OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4723/01**

MATHEMATICS

Core Mathematics 3

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Functions f and g are defined for all real values of x by

$$f(x) = x^3 + 4 \quad \text{and} \quad g(x) = 2x - 5.$$

Evaluate

(i) $\text{fg}(1)$, [2]

(ii) $\text{f}^{-1}(12)$. [3]

2 The sequence defined by

$$x_1 = 3, \qquad x_{n+1} = \sqrt[3]{31 - \tfrac{5}{2}x_n}$$

converges to the number α.

(i) Find the value of α correct to 3 decimal places, showing the result of each iteration. [3]

(ii) Find an equation of the form $ax^3 + bx + c = 0$, where a, b and c are integers, which has α as a root. [3]

3 **(a)** Solve, for $0° < \alpha < 180°$, the equation $\sec \frac{1}{2}\alpha = 4$. [3]

(b) Solve, for $0° < \beta < 180°$, the equation $\tan \beta = 7 \cot \beta$. [4]

4 Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

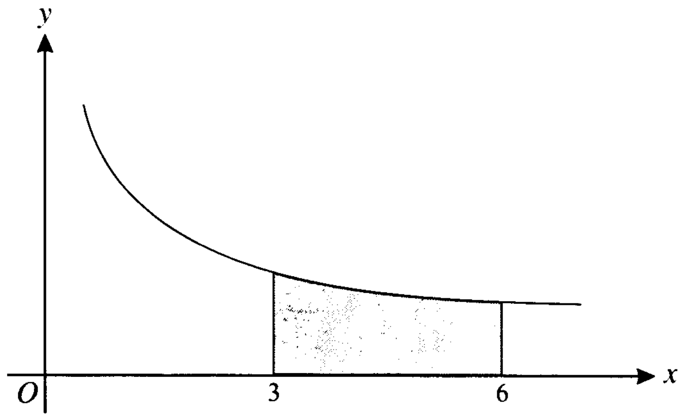
$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

(i) Find the value of $\frac{dV}{dh}$ when $h = 2$. [3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures. [3]

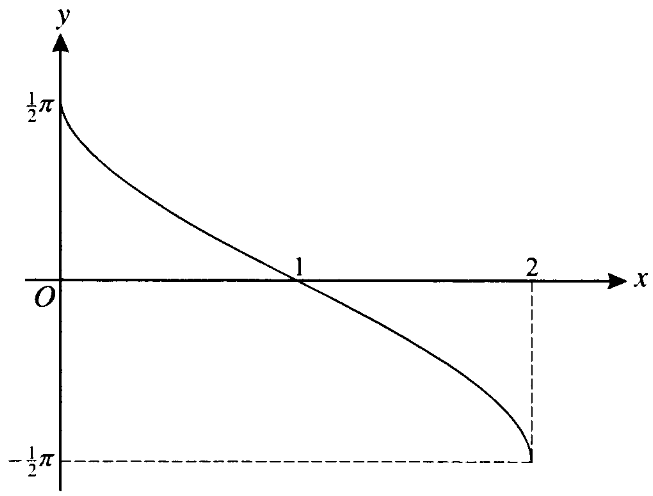
5 **(a)** Find $\int (3x+7)^9 \, dx$. [3]

(b)



The diagram shows the curve $y = \dfrac{1}{2\sqrt{x}}$. The shaded region is bounded by the curve and the lines $x = 3$, $x = 6$ and $y = 0$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced, simplifying your answer. [5]

6



The diagram shows the graph of $y = -\sin^{-1}(x-1)$.

(i) Give details of the pair of geometrical transformations which transforms the graph of $y = -\sin^{-1}(x-1)$ to the graph of $y = \sin^{-1} x$. [3]

(ii) Sketch the graph of $y = |-\sin^{-1}(x-1)|$. [2]

(iii) Find the exact solutions of the equation $|-\sin^{-1}(x-1)| = \frac{1}{3}\pi$. [3]

7 A curve has equation $y = \dfrac{xe^{2x}}{x+k}$, where k is a non-zero constant.

(i) Differentiate xe^{2x}, and show that $\dfrac{dy}{dx} = \dfrac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$. [5]

(ii) Given that the curve has exactly one stationary point, find the value of k, and determine the exact coordinates of the stationary point. [5]

8 The definite integral I is defined by

$$I = \int_0^6 2^x \, dx.$$

(i) Use Simpson's rule with 6 strips to find an approximate value of I. [4]

(ii) By first writing 2^x in the form e^{kx}, where the constant k is to be determined, find the exact value of I. [4]

(iii) Use the answers to parts **(i)** and **(ii)** to deduce that $\ln 2 \approx \frac{9}{13}$. [2]

9 **(i)** Use the identity for $\cos(A + B)$ to prove that

$$4\cos(\theta + 60°)\cos(\theta + 30°) \equiv \sqrt{3} - 2\sin 2\theta. \quad [4]$$

(ii) Hence find the exact value of $4\cos 82.5° \cos 52.5°$. [2]

(iii) Solve, for $0° < \theta < 90°$, the equation $4\cos(\theta + 60°)\cos(\theta + 30°) = 1$. [3]

(iv) Given that there are no values of θ which satisfy the equation

$$4\cos(\theta + 60°)\cos(\theta + 30°) = k,$$

determine the set of values of the constant k. [3]

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1 (i)	Show correct process for composition of functions	M1		numerical or algebraic; the right way round
	Obtain (–3 and hence) –23	A1	2	
(ii)	Either: State or imply $x^3+4=12$	B1		
	Attempt solution of equation involving x^3	M1		as far as $x = \ldots$
	Obtain 2	A1	3	and no other value
	Or: Attempt expression for f^{-1}	M1		involving x or y; involving cube root
	Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$	A1		
	Obtain 2	A1	(3)	and no other value
2 (i)	Obtain correct first iterate 2.864	B1		or greater accuracy 2.864327…; condone 2 dp here and in working
	Carry out correct iteration process	M1		to find at least 3 iterates in all
	Obtain 2.877	A1	3	after at least 4 steps; answer required to exactly 3 dp
	[3 → 2.864327 → 2.878042 → 2.876661 → 2.876800]			
(ii)	State or imply $x=\sqrt[3]{31-\frac{5}{2}x}$	B1		
	Attempt rearrangement of equation in x	M1		involving cubing and grouping non-zero terms on LHS
	Obtain equation $2x^3+5x-62=0$	A1	3	or equiv with integers
3 (a)	State correct equation involving $\cos\frac{1}{2}\alpha$	B1		such as $\cos\frac{1}{2}\alpha=\frac{1}{4}$ or $\frac{1}{\cos\frac{1}{2}\alpha}=4$ or …
	Attempt to find value of α	M1		using correct order for the steps
	Obtain 151	A1	3	or greater accuracy; and no other values between 0 and 180
(b)	State or imply $\cot\beta=\frac{1}{\tan\beta}$	B1		
	Rearrange to the form $\tan\beta=k$	M1		or equiv involving $\sin\beta$ only or $\cos\beta$ only; allow missing ±
	Obtain 69.3	A1		
	Obtain 111	A1	4	or greater accuracy; and no others between 0 and 180
4 (i)	Obtain derivative of form $kh^5(h^6+16)^n$	M1		any constant k; any $n<\frac{1}{2}$; allow if – 4 term retained
	Obtain correct $3h^5(h^6+16)^{-\frac{1}{2}}$	A1		or (unsimplified) equiv; no –4 now
	Substitute to obtain 10.7	A1	3	or greater accuracy or exact equiv
(ii)	Attempt multn or divn using 8 and answer from (i)	M1		
	Attempt 8 divided by answer from (i)	M1		
	Obtain 0.75	A1√	3	or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5	**(a)**	Obtain integral of form $k(3x+7)^{10}$	**M1**		any constant k
		Obtain (unsimplified) $\frac{1}{10}\times\frac{1}{3}(3x+7)^{10}$	**A1**		or equiv
		Obtain (simplified) $\frac{1}{30}(3x+7)^{10}+c$	**A1**	**3**	
	(b)	State $\int \pi(\frac{1}{2\sqrt{x}})^2\,dx$	**B1**		or equiv involving x; condone no dx
		Integrate to obtain $k\ln x$	**M1**		any constant k involving π or not; or equiv such as $k\ln 4x$ or $k\ln 2x$
		Obtain $\frac{1}{4}\pi\ln x$ or $\frac{1}{4}\ln x$ or $\frac{1}{4}\pi\ln 4x$ or $\frac{1}{4}\ln 4x$ **A1**			
		Show use of the log a – log b property	**M1**		not dependent on earlier marks
		Obtain $\frac{1}{4}\pi\ln 2$	**A1**	**5**	or similarly simplified equiv
6	**(i)**	Either: Refer to translation and reflection	**B1**		in either order; allow clear equivs
		State translation by 1 in negative x-direction	**B1**		or equiv but now using correct terminology
		State reflection in x-axis	**B1**	**3**	using correct terminology
		Or: Refer to translation and reflection	**B1**		in either order; allow clear equivs
		State reflection in y-axis	**B1**		
		State translation by 1 in positive x-direction	**B1**	**(3)**	with order reflection then translation clearly intended
	(ii)	Show sketch with attempt at reflection of 'negative' part in x-axis	**M1**		and curve for $0<x<1$ unchanged
		Show (more or less) correct sketch	**A1**	**2**	with correct curvature
	(iii)	Attempt correct process for finding at least one value	**M1**		as far as $x = \ldots$; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$
		Obtain $1-\frac{1}{2}\sqrt{3}$	**A1**		or exact equiv
		Obtain $1+\frac{1}{2}\sqrt{3}$	**A1**	**3**	or exact equiv; give **A1A0** if extra incorrect solution(s) provided
7	**(i)**	Attempt use of product rule for $x e^{2x}$	**M1**		obtaining … + …
		Obtain $e^{2x}+2xe^{2x}$	**A1**		or equiv; maybe within QR attempt
		Attempt use of quotient rule	**M1**		with or without product rule
		Obtain unsimplified $\frac{(x+k)(e^{2x}+2xe^{2x})-xe^{2x}}{(x+k)^2}$	**A1**		
		Obtain $\frac{e^{2x}(2x^2+2kx+k)}{(x+k)^2}$	**A1**	**5**	**AG**; necessary detail required
	(ii)	Attempt use of discriminant	**M1**		or equiv
		Obtain $4k^2-8k=0$ or equiv and hence $k=2$	**A1**		
		Attempt solution of $2x^2+2kx+k=0$	**M1**		using their numerical value of k or solving in terms of k using correct formula
		Obtain $x=-1$	**A1**		
		Obtain $-e^{-2}$	**A1**	**5**	or exact equiv

8 **(i)**	State or imply $h = 1$	**B1**		
	Attempt calculation involving attempts at y values	**M1**		addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values
	Obtain $a(1 + 4\times2 + 2\times4 + 4\times8 + 2\times16 + 4\times32 + 64)$	**A1**		any constant a
	Obtain 91	**A1**	**4**	
(ii)	State $e^{x\ln 2}$ or $k = \ln 2$	**B1**		allow decimal equiv such as $e^{0.69x}$
	Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$	**M1**		any constant k or in terms of general k
	Obtain $\frac{1}{\ln 2}(e^{6\ln 2} - e^0)$	**A1**		or exact equiv
	Simplify to obtain $\frac{63}{\ln 2}$	**A1**	**4**	allow if simplification in part (iii)
(iii)	Equate answers to (i) and (ii)	**M1**		provided ln 2 involved other than in power of e
	Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$	**A1**	**2**	**AG**; necessary correct detail required
9 **(i)**	State at least one of $\cos\theta\cos 60 - \sin\theta\sin 60$ and $\cos\theta\cos 30 - \sin\theta\sin 30$	**B1**		
	Attempt complete multiplication of identities of form $\pm\cos\cos \pm \sin\sin$	**M1**		with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved
	Use $\cos^2\theta + \sin^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta$	**M1**		
	Obtain $\sqrt{3} - 2\sin 2\theta$	**A1**	**4**	**AG**; necessary detail required
(ii)	Attempt use of 22.5 in right-hand side	**M1**		
	Obtain $\sqrt{3} - \sqrt{2}$	**A1**	**2**	or exact equiv
(iii)	Obtain 10.7	**B1**		or greater accuracy; allow ±0.1
	Attempt correct process to find two angles	**M1**		from values of 2θ between 0 and 180
	Obtain 79.3	**A1**	**3**	or greater accuracy and no others between 0 and 90; allow ±0.1
(iv)	Indicate or imply that critical values of $\sin 2\theta$ are -1 and 1	**M1**		
	Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$	**A1**		condoning decimal equivs, $\le \ge$ signs
	Obtain complete correct solution	**A1**	**3**	now with exact values and unambiguously stated